



Semester Two Examination, 2020

Question/Answer booklet

**MATHEMATICS
METHODS
UNIT 3&4**

**Section One:
Calculator-free**

If required by your examination administrator, please
place your student identification label in this box

WA student number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
Total					100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free

35% (52 Marks)

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1

(7 marks)

(a) Determine an expression for $f'(x)$ when

(i) $f(x) = \ln(1 - \cos 3x)$.

(2 marks)

(ii) $f(x) = e^{5x}(5 - 2x)^3$.

(3 marks)

(b) For the positive number x , let $A(x) = \int_0^x (8 - 2t^2) dt$.

Determine the value(s) of x for which $\frac{dA}{dx} = 0$.

(2 marks)

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Question 2

(5 marks)

The rate of change of pressure in an air tank is given by $P'(t) = -3e^{-0.05t}$, where t is the time in minutes since it began emptying from an initial pressure of 70 psi.

(a) Determine an expression for the pressure P in the tank at any time t , $t \geq 0$. (2 marks)

(b) Determine

(i) the time taken for the pressure in the tank to fall to 40 psi. (2 marks)

(ii) the minimum pressure in the tank for $t \geq 0$. (1 mark)

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Question 3

(6 marks)

The continuous random variable X takes values in the interval 1 to 5 and has cumulative distribution function $F(x)$ where

$$F(x) = P(X \leq x) = \begin{cases} 0 & x < 1 \\ \frac{x-1}{4} & 1 \leq x \leq 5 \\ 1 & x > 5. \end{cases}$$

(a) Determine

(i) $P(X \leq 3.5)$. (1 mark)

(ii) the value of k , if $P(X > k) = 0.85$. (2 marks)

(b) Determine $f(x)$, the probability density function of X , and sketch the graph of $y = f(x)$. (3 marks)

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Question 4

(7 marks)

The function f is defined by $f(x) = \frac{x^2 - 5}{3 - x}$, $x \neq 3$.

The second derivative of f is $f''(x) = 8(3 - x)^{-3}$.

Determine the coordinates and nature of all stationary points of the graph of $y = f(x)$.

Question 5

(7 marks)

(a) Simplify $\log 8 + 2 \log 5 - \log 2$.

(2 marks)

(b) Given that $\log_a x = 1.4$, determine the value of $\log_a x\sqrt{x}$.

(2 marks)

(c) Determine the solution to the equation $5^{2x} = 2^{3-x}$ in the form $x = \frac{\log a}{\log b}$.

(3 marks)

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Question 6

(7 marks)

The discrete random variable X is defined by

$$P(X = x) = \begin{cases} \frac{x + k}{3x + 2} & x = 0, 1 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Determine the value of the constant k .

(2 marks)

(b) Determine

(i) $E(X)$.

(1 mark)

(ii) $E(3X - 1)$.

(2 marks)

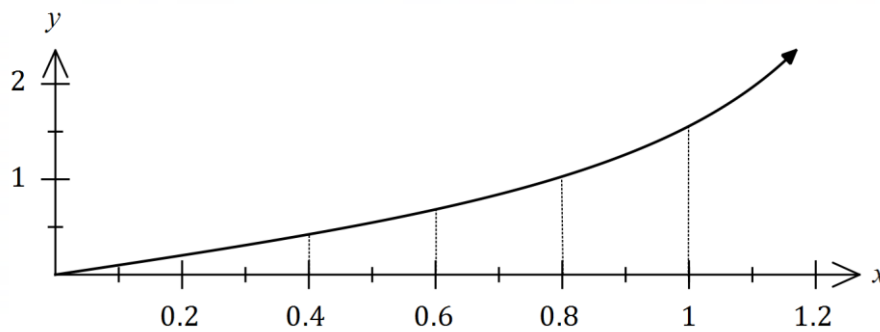
(c) Determine $\text{Var}(3X - 1)$.

(2 marks)

Question 7

(7 marks)

The graph and a table of values for $y = f(x)$ is shown below, where $f(x) = \tan x$.



x	y
0.2	0.2
0.4	0.42
0.6	0.68
0.8	1.03
1	1.56
1.2	2.57

Let $I = \int_{0.4}^1 \tan x \, dx$.

- (a) By using the information shown and considering sums of the form $\sum_i f(x_i)\delta x_i$, explain why $I > 0.426$. (3 marks)
- (b) In a similar manner to (a), determine the best estimate for the value of the constant U , where $I < U$. (2 marks)
- (c) Use your previous answers to determine a numerical estimate for I and explain whether your estimate is smaller or larger than the exact value of I . (2 marks)

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Question 8

(6 marks)

The acceleration at time t seconds of a small body travelling in a straight line is given by

$$a(t) = \frac{-3}{\sqrt{2t+3}} \text{ cm/s}^2, \quad t \geq 0.$$

When $t = 3$ the body was at the origin and 8 seconds later its displacement was 30 cm.

Determine the velocity of the body when $t = 6.5$.

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Supplementary page

Question number: _____

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